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The collapse of gas discs in non-axisymmetric galaxy cores

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ABSTRACT
Below a threshold energy, gas in the constant density core of a triaxial galaxy can find no simple non-intersecting periodic orbit to act as an attractor for its trajectory (El-Zant et al. 2003). If a disc of gas arriving from further out in the galaxy dissipates sufficient energy to fall below this threshold, it will thereafter collapse into the very centre. Such a mechanism may be relevant to the early growth of super-massive black holes at the Eddington limit and the appearance of the quasar phenomenon at high redshift. This process is self-limiting in the sense that, when the black hole mass has grown to a significant fraction of the core mass, simple angular momentum conserving orbits are restored and accretion reverts to the slow viscous mode. The mechanism depends upon the pre-existence of constant density cores in triaxial spheroidal galaxies.

Key words: Galaxies– active, galaxies– gas flow, galaxies– orbit structure

1 INTRODUCTION

It is now evident that the nuclei of most, if not all, spheroidal galaxies contain super-massive black holes (SBH). The fact that the phenomenon of extreme activity in galactic nuclei– the quasar phenomenon– peaked at early cosmic epochs and declined subsequently (e.g. Wall et al. 2005) suggests that the construction of massive black holes in galaxies was coeval with galaxy formation itself– that black hole formation is an integral part of the galaxy formation process. The tight correlation between black hole mass and the galaxy stellar velocity dispersion– the $M_h - \sigma$ relation– (Ferrarase & Merritt 2000; Gebhardt et al. 2000), which requires a close connection between the black hole and the larger scale galaxy kinematics, would seem to support this idea of simultaneous growth.

An alternative (and older) viewpoint is that galaxies, and the corresponding deep potential wells, were in place essentially before the black holes began their rapid growth– that preexisting galactic centres defined the sites for SBH formation rather than vice versa (see Merritt, 2006, for a recent summary and a more complete list of references). If true, then the epoch of galaxy formation must be further back in cosmic history ($z \geq 10$) than the appearance of the quasar phenomenon. The recent discovery of high redshift galaxies (Iye et al. 2006; Eyles et al. 2006) would be consistent this view. The $M_h - \sigma$ scaling relation would then appear to require that some aspect of galaxy structure should both promote and then limit the growth of the black hole. This is the possibility that I will explore here in the context of a specific mechanism for supplying the matter necessary for black hole growth.

The mass source necessary for the rapid growth of nuclear black holes– the construction material– and its transport to the near vicinity of the event horizon have always been problematic. Of course, an important mechanism for the growth to high mass could well be the merging of preexisting black holes due to galaxy mergers. However, the quasar phenomenon itself and the contribution of active nuclei to the X-ray background (Elvis, Risaliti & Zamorani 2002; Fabian & Iwasawa 1999) would seem to require that a substantial fraction of the present mass content in SBHs is due growth from lower mass seeds via accretion of gas at the Eddington limit, with a high efficiency of producing electromagnetic radiation.

Diffuse gas in spheroidal galaxies is an obvious building material for super-massive black holes. It may arrive in the central regions of a galaxy from either internal or external sources (Shlosman, Begelman & Frank 1990), and unlike stars, gas is dissipative; through energy loss, the gas can sink deeper into the potential well. However, for the Eddington growth of black holes, rapid loss of angular momentum is a more significant problem. To arrive at the core of a galaxy (at radii of 10-1000 pc) gas initially in equilibrium further out (at roughly an effective radius) must loose typically 90% of its angular momentum. This appears to be possible due to the effects of non-axisymmetric distortions of the potential– intrinsic bars, or bars excited during encounters and mergers (e.g. Sellwood & Moore 1999).

But an even more significant problem is transport from the galaxy core to the Schwarzschild radius, where the gas must reduce its angular momentum by a factor of $10^7 - 10^8$. The viscous inflow timescale in a classical accretion disc with subsonic turbulence appears to be much too long to fuel an extended period of high-luminosity activity (Shlosman, Begelman & Frank 1990). Therefore, here

I consider a simple mechanism which may bridge this final, but significant, gap. To work, this mechanism requires two assumptions: first, that spheroidal systems, such as elliptical galaxies or bulges of spiral galaxies, initially contain a constant density core at their centres; and second, that these cores are mildly non-axisymmetric.

The first assumption is contentious. The prevailing viewpoint at present is that power law cusps form naturally in dissipationless collapse and constant density cores may be created later by the "scouring" action of binary SBHs (i.e., gravitational scattering of stars by two in-spiralling black holes (Mirosavljvić & Merritt 2001)). This idea is supported by the fact that higher luminosity spheroidal systems, those in which merging of equal partners has probably played a more dominant role, often appear to contain constant density cores; whereas, lower mass systems, such as the Milky Way bulge, are power law ($r^{-1.5}$) into small radii (Faber et al. 1997; Merritt 2006). On the other hand, evolution could go the other way: Cores may originally be present in galactic nuclei and then altered by the growth of the SBH and the resulting adiabatic or diffusive formation of a cusp in the stellar density distribution (Peebles 1972; Bahcall & Wolf 1976; Young 1980). The appearance of the original core may also be altered as a consequence of subsequent star formation via processes such as the one I will consider here.

The second assumption is less controversial. For some years now it has been appreciated that slowly rotating but non-spherical hot stellar systems are supported by an anisotropic velocity distribution. This provides naturally a triaxial system with a non-rotating figure (Schwarzschild 1979). Here we require a gravitational potential which is mildly non-axisymmetric in a principal plane. This asymmetry must extend into the core, and, indeed, various indicators of non-axisymmetric structures are actually observed in galactic nuclei (Shaw et al. 1995).

Given these two assumptions, the mechanism is simple: a gas disc is supported against gravity by motion on near-circular orbits– orbits which serve as the parents of the tube families. But within a constant density non-axisymmetric core there are no tube orbits below a critical orbital energy; there are only box orbits which, after sufficient time, pass arbitrarily close to the centre (see e.g. Binney & Tremaine 1987). That is to say, within such a core, orbital angular momentum– or even a sense of rotation– is not conserved; the two integrals or motion are effectively the oscillation periods along the principal axes, and, if these are non-commensurate, the orbit fills an elongated box after infinite time. Then the gas, being dissipative, accumulates at the centre.

This idea is not new. It was originally suggested by Lake & Norman (1983) in a wide-ranging paper discussing the orbit structure in triaxial systems and the relationship between that orbit structure and gas flow. At about this time it was appreciated that simple non-self-intersecting periodic orbits act as attractors for gas flow in non-axisymmetric systems, and much of the structure in gas-rich galaxies can be understood in the context of this fact (Sanders, Teuben & van Albada 1983). Lake and Norman realized that if there are no simple periodic orbits over some range of energy– that if there is no integral of motion preserving a sense of rotation like angular momentum– then the ultimate attractor is the centre of the galaxy, and one might expect gas inflow to be significantly enhanced.

The inability of a gas disc to be sustained in a constant density non-axisymmetric core is the central aspect of a model by El-Zant et al. (2003) for the simultaneous formation of a SBH and axisymmetric spheroid in the presence of a triaxial CDM halo. A constant density core is presumably created in the cuspy CDM halo by the scouring action of baryonic clumps (El-Zant, Shlosman & Hoffman 2001). The re-emergence of tube orbits in the increasingly axisymmetric potential would then limit the growth of both the SBH and the spheroid and, with additional assumptions, explain the observed $M_h - \sigma$ relation. Here I take the standpoint that the baryonic component of a spheroidal galaxy is essentially in place when the black hole begins to grow, and that a dark halo plays negligible role in this process. There is ample evidence that the spheroid itself is triaxial and that the baryonic components are completely dominant within an effective radius (Trimblay & Merritt 1995; Romanowsky et al. 2003).

I consider the details of gas disk collapse in triaxial galaxy cores via "sticky particle" calculations. The first problem is that of energy loss; low angular momentum gas entering the vicinity of core must dissipate sufficient energy to fall below the threshold for the disappearance of tube orbits. I model this process by an in-falling gas annulus, with angular velocity insufficient to balance gravity; the annulus falls past an equilibrium point and oscillates radially inward and outward. In multiple bounces over several dynamical timescales, energy is dissipated until a significant fraction of the gas has penetrated the critical energy below which tube orbits do not exist. The gas disc then collapses to the centre within one or two dynamical timescales.

Such a mechanism could not only lead to fuelling of low mass seed black holes at a rate near the Eddington limit, but, as stressed by El-Zant et al. (2003), it is also self-limiting. When the black hole mass grows to a substantial fraction of the core mass, tube orbits reappear in the core and the box orbits become chaotic (Merritt 2006). Since gas is preferentially trapped on non-chaotic orbits, a more typical gas disc re-emerges with slow, viscosity-driven accretion onto the SBH. If core properties are closely tied to the overall properties of the stellar system, the global scaling relations for SBHs might be explained. But it is important to appreciate that the mechanism described here can only be relevant to the early growth of black holes– at the epoch of galaxy assembly– and not to present activity associated with SBHs in galactic nuclei.

2 ORBIT STRUCTURE AND GAS FLOW IN TRIAXIAL CORES

Here, following Lake & Norman 1983 and El-Zant et al. 2003, I review the relevance of orbit structure to gas motion in triaxial systems. I assume that the spheroidal galaxy, in a principle plane, is described by the potential

$$\Phi(x, y) = \frac{{V_o}^2}{2} \ln\Big({r_c}^2 + x^2 + \frac{y^2}{q^2}\Big) \tag{1}$$

where $q \leq 1$ (Binney & Tremaine 1987). This logarithmic potential contains a simple harmonic core and would, in the

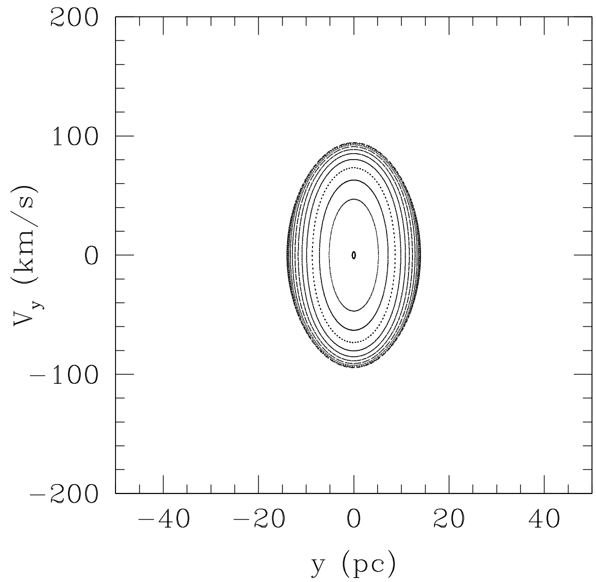

Figure 1. Surface of section ($y - \dot{y}$ plane) for an energy of 1.41×10^5 (km/s)^2 in the potential given by eq. 1. This is sufficient to carry particles to a radius of about 0.5 r_c. The point at the origin is the long axis radial orbit and the invariant curves represent boxes librating about this orbit. This deep in the core, there are no loop orbits.

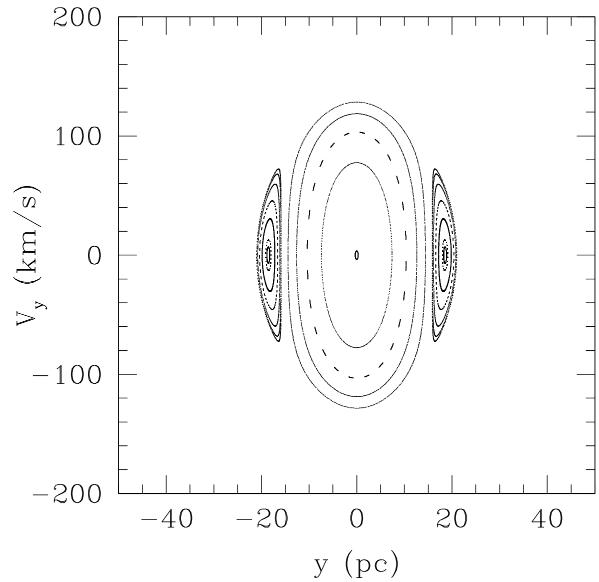

Figure 2. Surface of section ($y - \dot{y}$ plane) for an energy of 1.45×10^5 (km/s)^2 in the potential given by eq. 1. This is sufficient to carry particles to a radius of about 0.75 r_c. Again we see the family of box orbits surrounding the long axis radial orbit at the origin, but at this energy invariant curves representing two families of loop orbits, corresponding to different senses of circulation, have also appeared.

case of axial symmetry ($q = 1$), provide a system with a flat rotation curve (rotation velocity V_o) beyond the core. This corresponds to a constant density core with a density distribution asymptotically approaching $1/r^2$ beyond a core radius r_c (similar to an isothermal sphere). The central density is given by

$$\rho(0) = \frac{3{V_o}^2}{4\pi G{r_c}^2}. \qquad (2)$$

Here, as a numerical example, I take "typical" elliptical galaxy values of $r_c = 30$ pc and $V_o = 200$ km/s. In this case, $\rho(0) = 2.46 \times 10^3$ $M_\odot/pc^3$ and the total stellar mass of the core (mass out to r_c) would be about 1.5×10^8 $M_\odot$. In the calculations described below I take $q = 0.937$– a mild deviation from axial symmetry avoiding commensurable oscillations along the x and y axes.

The orbital structure in such a system can be understood by considering surfaces of section– the maps generated by progressive penetrations of a plane in the four dimensional phase space by orbits at a given energy. Fig. 1 is such a surface of section on the y-$\dot{y}$ plane for orbits at an energy of 1.405×10^5 (km/s)^2 – sufficient to carry the particles to a maximum radius of 0.5 r_c (15 pc in this case); i.e., these would be orbits fairly deep within the core. The point at the centre is the long axis periodic orbit (radial oscillations along the x axis). It is evident that all surrounding curves represent orbits which librate about this long axis radial orbit with no preferred sense of rotation; there is no integral of motion analogous to angular momentum, and there are no loop orbits this deep in the core.

Fig. 2 is a surface of section at a higher energy of 1.45×10^5 (km/s)^2, sufficient to carry a particle to a radius of 0.75 r_c. By this point the loop orbits, evident as the two sets of closed curves beyond the box orbits, have developed. These two sets correspond to opposite senses of rotation. The invariant curves close about two periodic orbits which are slightly elongated perpendicular to the major axis of the potential distribution. The loops librate about these periodic orbits. Steady state gas flow streamlines would be expected to correspond to one of these two families of closed periodic orbits– the parents of the loop families– depending upon the sense of rotation.

At energies lower than $E_T = 1.42 \times 10^5$ (km/s)^2, intermediate between the two cases shown and sufficient to carry particles out to a radius of 0.6 r_c, there are no loops; i.e., E_T represents a threshold above which loops are found for this particular value of q (for smaller q the threshold is at higher energy). What, then, is the fate of gas that diffuses below E_T? Angular momentum, or at least the integral I_2 which becomes angular momentum in the limit of axial symmetry, is no longer conserved; the only orbits available are the boxes, and we would expect the gas to collapse to the centre on a dynamical timescale.

The expectation is altered, however, by the presence of a SBH at the centre of the core. Adding the potential of a point mass ($-GM_h/r$) to eq. 1, where the mass of the black hole, M_h, is a substantial fraction of the core mass, causes the re-appearance of loop orbits and the re-emergence of the angular momentum-like integral of motion. This is illustrated in Fig. 5 which is a surface of section at an energy sufficient to carry particle to a radius of $0.6r_c$ when a point mass of 5×10^7 $M_\odot$ (about 1/3 of the core mass) has been placed at the centre of the system described above . We see now the

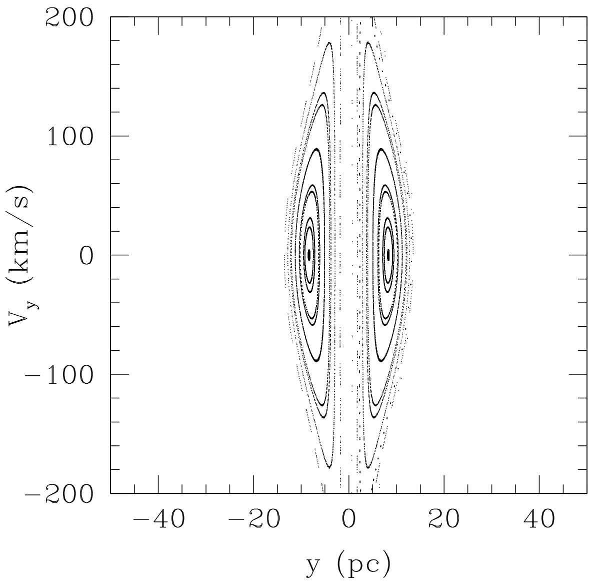

Figure 3. Surface of section ($y - \dot{y}$ plane) in the potential given by eq. 1 to which a point mass of 5×10^7 $M_\odot$ has been added. Here the energy is 1.25×10^5 (km/s)^2 which, as in the case shown in Fig. 1, is sufficient to carry particles to a radius of about 0.5 r_c. Here the loop orbits have reappeared deep in the core, and some fraction of the box orbits have become chaotic.

presence of two families of loops. Some fraction of the box orbits become chaotic, but in most contexts, gas motion in the presence of both chaotic and non-intersecting periodic orbits is trapped on the periodic orbits (Lake & Norman 1983; Sanders, Teuben & van Albada 1983). This suggests that the growth of the black hole by this process of rapid gas accretion in the core would be self-limiting. The reappearance of circulating periodic orbits forces the flow back to the slow, viscous accretion mode and cuts off the possibility of further growth at the Eddington limit. These expectations are supported by sticky particle calculations described in the following section.

3 STICKY PARTICLE CALCULATIONS

The technique for including dissipation in particle motion is essentially that which I have previously applied to simulating Galactic Centre clouds (Sanders 1998). One assumes an interaction distance σ such that initially each particle has about 10 neighbours within this distance. Then at each time step every particle adjusts its velocity so as to reduce the velocity difference with these close neighbours, but only if that velocity difference is negative (i.e., if the particle are approaching each other).

If $\mathbf{V_{ij}}$ is the component of the relative velocity along the line joining the two particles (i.e., the vector at the position of particle i pointing away from particle j), then the velocity of particle i during time-step Δt_k changes by

$$\delta \mathbf{v_{ik}} = \alpha_k \sum_j^{r_{ij}<\sigma} \mathbf{V_{ij}} \tag{4}$$

where

$$\alpha_k = \Delta t_k / \Delta t_d \tag{5}$$

and Δt_d is an adjustable dissipation timescale. This provides, in effect, a bulk velocity in which every particle's velocity is adjusted proportionally to the local velocity divergence, but only if that divergence is negative. The algorithm conserves linear momentum and, in axial symmetry, the angular momentum of an ensemble of particles, but obviously does not conserve energy. In the following examples I take $\sigma = 0.6$ pc and $\Delta t_d = 0.02$ dynamical timescales (20000 years).

For the mechanism of trapping on box orbits to work, the in-falling gas disc must not just penetrate to within about 0.5 r_c, it must also dissipate sufficient energy to fall below the threshold E_T below which no loops are present. Therefore, as an initial condition I take 4000 "gas" particles to be uniformly distributed between radii of 20 pc and 40 pc, in pure tangential motion about the centre, but with only 40 % of the velocity required to balance gravity; i.e., the centripetal acceleration is 0.16 of the gravitational acceleration. In addition I give the particles a random motion of about 10 % of the tangential velocity (10-20 km/s). This initial condition is arbitrary, but could correspond to low angular momentum gas flowing into the core from larger radii– either as a result of a stellar merger or as gas lost from stars during normal stellar evolution (it would seem quite unlikely that gas would arrive in the vicinity of the core with zero specific angular momentum). In any case, because the gas comes from further out in the system, it must dissipate energy before the mechanism I describe can work.

Given this initial condition I consider in-fall in three different variants of the potential given by eq. 1: a) The potential is axisymmetric with $q = 1$; b) There is a mild asymmetry with $q = 0.937$ as for the orbits shown by surface-of-section in Figs. 1-2; c) the potential of a point mass of 5×10^7 $M_\odot$ has been included in the non-axisymmetric case as for the surface of section shown in Fig. 3. The initial distribution of the gas particles and the final distributions, after eight characteristic orbit times (8 million years) are shown in Fig. 4.

Case *a*, inflow in the axisymmetric potential, is interesting because it demonstrates how fairly rapid dissipation of energy can take place. After eight orbit times we see that a ring has formed at mean radius of about 18 pc. This ring oscillates initially with large radial excursions but the oscillations damp away within a few dynamical timescales due to the effective dissipation inherent in this sticky particle routine. The average orbital energy per particle is shown in Fig. 5 (dashed line) where it is evident that the energy decreases in steps. These steps down occur when the ring is at its minimum radius– at the bounce. Here the inner part of the ring is moving outward while the outer part is still moving inward; hence there is large compression and dissipation. The specific angular momentum (average per particle) as a function of time is shown in Fig. 6, and it is clearly conserved to high precision.

In the mildly non-axisymmetric case *b*, the simulated gas annulus has collapsed to the centre within eight rotation periods. This is because of the rapid dissipation of energy in radial oscillations– as is evident in Fig. 5 where again we see the pronounced steps down in energy corresponding to

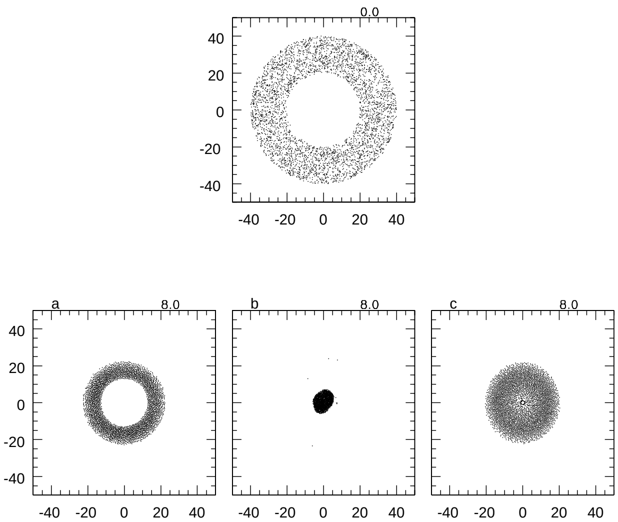

Figure 4. The top figure show the initial distribution of 4000 sticky particles for the three cases considered. The particles are uniformly distributed between 20 pc and 40 pc but with only 16% of the centripetal force required to balance gravity. The final distributions, after eight orbital timescales (8 million years) are shown below for: a) the axisymmetric potential ($q = 0$); b) the non-axisymmetric potential ($q = 0.937$); c) the non-axisymmetric potential to which a central point mass of 5×10^7 $M_\odot$ has been added. The axes are labelled in pc.

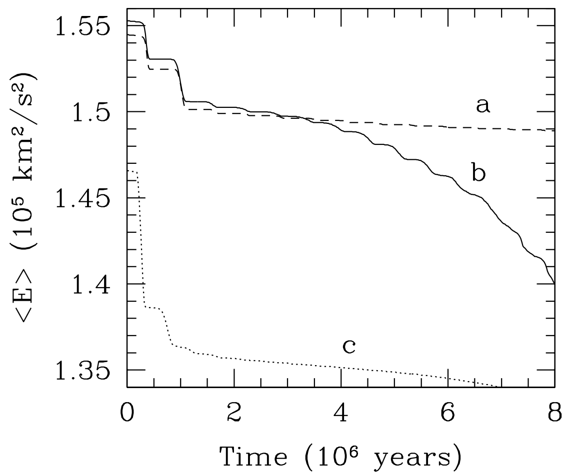

Figure 5. The average orbital energy per particle (in 10^5 km^2/s^2) as a function of time (10^6 years) for the axisymmetric case a (dashed line) and the non-axisymmetric case b (solid line), and the non-axisymmetric case with a central black hole of 5.0×10^7 $M_\odot$, case c. It is evident that the energy decreases in steps corresponding to maximum compression. By this process, gas particles penetrate the energy threshold below which tube orbits disappear and this leads to the rapid collapse of the gas disc in case b.

minimum contraction of the non-axisymmetric gas ring. By five or six rotation periods, most of the gas particles have penetrated below E_T and entered the inner part of the core where tube orbits no longer exist. The angular momentum, no longer conserved, then decreases to less than 0.1 of its original value (Fig. 6). Of course, the total angular momentum of the entire system of stars and gas must be conserved. This means that the original angular momentum of the gas disk is is lost to the non-axisymmetric stellar system via torques. This would tend to give the trixial system a figure rotation, or, more likely, would result in a heating of the system and, on the long term, a restoration of axial symmetry in the core.

In case c, where the black hole has been added, the infalling annulus has become a standard accretion disc which very slowly drains into the hole. This is due to the re-emergence of loop orbits (Fig. 3) deep within the core and the presence of a second angular momentum-like integral. Although there is a large dissipation of orbital energy in the first two bounces (Fig. 5), the angular momentum, after an initial small decrease, is well-conserved (Fig. 6).

The steps by which the gas disc in the non-axisymmetric case b collapses is broadly traced by the time sequence shown in Fig. 7. Here it is evident that an asymmetric ring is formed which, after oscillating in and out, gradually closes and collapses to the centre In fact, the time intervals in this figure are too large to show the details of the collapse; the asymmetric ring opens and closes several times before the final collapse, but it is clear that the collapse occurs over several dynamical timescales rather than on a slow viscous timescale.

During the collapse of the gas annulus, there is significant compression of the gas and resulting strong shocks. Therefore, star formation would probably proceed in such an environment. In case b, for example, maximum compression occurs at the point where the elliptical ring has reached its minimum radius, therefore we might expect star formation to occur in bursts corresponding to the steps down in energy (Fig. 5). Even stronger compression of the gas develops when the ring collapses to the centre between frames 5 and 6 in Fig. 7 because here a number of the gas particles are on a counter-rotating path (recall that the relevant orbits are boxes).

In the sticky particle technique for including dissipation (eq. 4) , the velocity divergence at time k at the position of particle i can be estimated:

$$-(\nabla \cdot v)_{ik} = \frac{|\Delta \mathbf{v}_{ik}|}{\alpha_k \sigma} \tag{5}$$

where $\Delta \mathbf{v}_{ik}$ is given by eq. 3. To simulate star formation I assume that if the compression defined by eq. 5 exceeds a certain threshold, the dissipation for that particle is turned off and the particle motion is thereafter only affected by gravity (as in Sanders 1998). Here I arbitrarily have set the compression threshold for star formation at 650 km/(s-pc); with a factor of two higher threshold relatively few particles are converted into stars, and with a factor of two lower threshold, the majority of the particles become stars. With this threshold, about 630 of the original 4000 gas particle have been converted to stars by the end of the simulation.

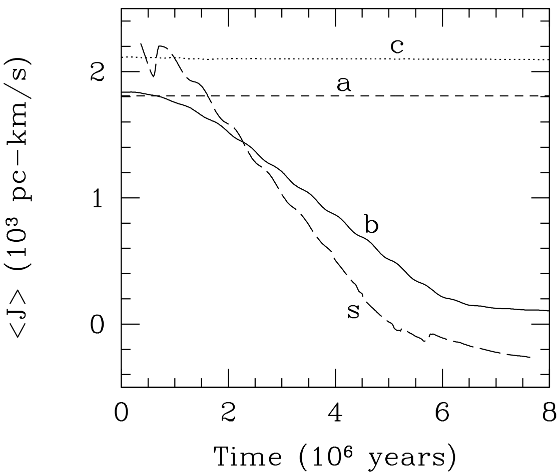

Figure 6. The average specific angular momentum (units of 10^3 pc-km/s) per particle as a function of time (orbit times or millions of years) for collapse cases *a*, *b* and *c*, as well as for newly formed stars in case *b* (labelled *s*). In the axisymmetric case *a* the angular momentum is conserved but in the non-axisymmetric case there is a dramatic decrease as the disc collapses to the centre. In the presence of a black hole the angular momentum is restored as a conserved integral. With respect to the newly formed stars, note that this is the average angular momentum per star; the angular momentum of the initially formed stars is not lost but is diluted as more stars are formed in the collapse. The ensemble of stars formed from the collapsing gas via the compression criterion (see eq. 5) is characterised, finally, by counter-rotation (negative average angular momentum).

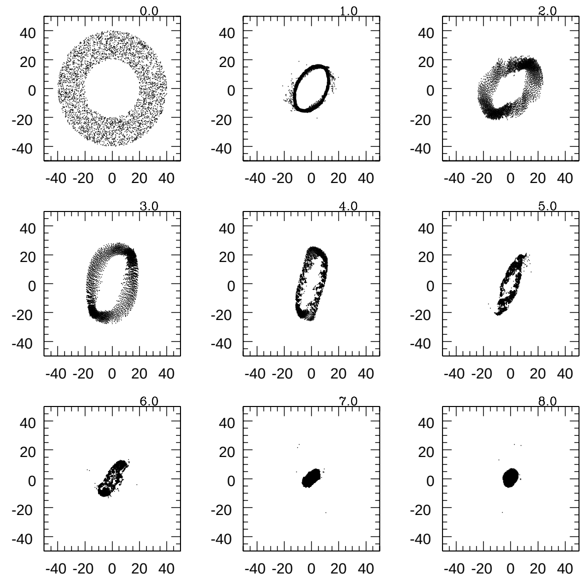

Figure 7. A time sequence showing the evolution of the in-falling gas ring in the mildly non-axisymmetric case *b*. The frames are separated by one orbit time or 1 million years (time labelled on upper right of each frame). Between frames 5 and 6 the ring completely closes and the resulting dissipation causes the gas to collapse to the centre.

The spatial distribution of these newly formed stars is shown in Fig. 8, which is the same time sequence as for the gas distribution (Fig. 7). As expected, the new stars form at maximum compression with a large burst during the final collapse of the annulus. It is of interest that the ensemble of stars formed by t=8, primarily in this final collapse, is counter-rotating with respect to the original gas disc; this is because those fluid elements which find themselves moving against the direction of most of the fluid experience the largest compression. The counter rotation is also evident from Fig. 6 where the curve labelled *s* shows the time dependence of average angular momentum per star for the newly created stars. By the end of the simulation it is negative. It is also of interest that the density of these newly formed stars increases toward the centre; this could create the appearance of a cusp in the presence of a constant density core. (for movies of all four simulations go to htpp://www.astro.rug.nl/∼ sanders/movie.html)

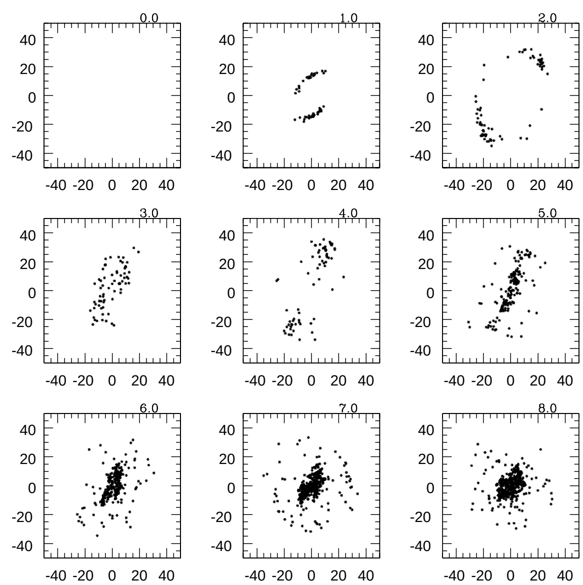

Figure 8. A time sequence showing the appearance and distribution of stars newly formed via compression in the mildly non-axisymmetric case *b*. The frames are separated by one orbit time or 1 million years and correspond exactly to the frames of the gas distribution shown in Fig. 7. Between frames 5 and 6, when the ring completely closes, the compression is maximum and most stars form.

4 A NOTE ON SCALING RELATIONS

Any model for the growth of black holes in galactic nuclei must address the issue of the surprisingly tight relation between the black hole mass and the larger scale velocity dispersion in the spheroidal galaxy; i.e.,

$$M_h \propto {V_s}^{\beta} \tag{6}$$

where β is in the range of 4 to 4.5 (Ferrarase & Merritt 2000; Gebhardt et al. 2000; Tremaine et al. 2002). This is possible in the context of the mechanism described here because it is self-limiting with the rapid growth of the SBH occurring until the hole mass becomes on the order of 10% to 20% of the core mass. A correlation between the core mass and the velocity dispersion is the remaining missing ingredient.

This may be possible if the formation of a core is viewed in terms of a maximum phase space density (Dalcanton & Hogan 2001). The average density in a core giving rise to the potential described by eq. 1 is

$$\bar{\rho_c} \approx \frac{2{V_s}^2}{2\pi G{r_c}^2} \tag{7}$$

which means that the average phase space density $\bar{\rho_c}/{V_s}^3$ is

$$f_c = \frac{3}{2\pi G V_s {r_c}^2} \tag{8}$$

Combining these relations (given that the core mass is $\rho_c {r_c}^3$) we find

$$M_c \approx \left(\frac{V_s}{G}\right)^{\frac{3}{2}} {f_c}^{-\frac{1}{2}}; \tag{9}$$

i.e., there is a built-in correlation between core mass and velocity dispersion.

It has been demonstrated that the average course-grained phase space density in galaxies decreases as galaxy luminosity increases (Hernquist, Spergel & Heyl 1993; Dalcanton & Hogan 2001). If spheroidal galaxies are more or less homologous then, combined with a luminosity-velocity dispersion relation (Faber-Jackson), this would imply $f_c \approx {V_s}^{-p}$ where p is between 4 and 6. Then combined with eq. 9 we have

$$M_h \approx 0.1 M_c \propto {V_s}^{(3+p)/2}$$

which could explain the observed relation eq. 6. This is all quite speculative and it is unclear that such correlations could account for the tightness of the $M_h - V_s$ relation.

It also entirely depends upon the pre-existence of cores in spheroidal galaxies– cores with properties set by phase space constraints. In pure CDM halos, of course, there are no cores because there are no phase space constraints, but there are clear observational indications that cores do exist at least in dwarf spirals (de Blok et al. 2001; Gentile et al. 2004). This issue remains to be settled.

5 SUMMARY AND SPECULATION

Observed constant density (or constant surface brightness) cores in early type galaxies range in radius from 10 to 1000 pc (Faber et al. 1997). Gas deposited in such cores from some exterior source, must collapse into the centre if the core is mildly non-axisymmetric with a non-rotating figure and not yet dominated by the central black hole. This collapse is due to the absence of simple closed, non-self-intersecting periodic orbits below a threshold energy; such orbits normally play the role of attractors in the phase space of the dissipational medium. In some inner fraction of the core, which can be significant even in the presence of a weak deviation from axial symmetry, there are only box orbits librating about the long and short axis radial orbits.

Once the gas, through dissipation, has penetrated below the threshold energy, collapse is rapid– occurring within a few dynamical timescales, and this could lead to Eddington growth of small seed black holes. Because, in such dynamical collapse, strong shocks develop in the gas with resulting high compression, some fraction of a collapsing gas disc may be turned into stars; the density of these new stars increases into the centre and may disguise the original constant density core. In this sense, it is interesting that bright nuclei are found in 29% of cored galxies and 60% of galaxies with central powerlaw cusps (Lauer et al. 2005), and that these cusp nuclei are significantly bluer than the surrounding galaxy.

The process is self-limiting in the sense that when the mass of a central black hole has grown to be 10% to 20% of the mass of the stellar core, the simple periodic orbits reappear within the core; angular momentum re-emerges as an integral of motion. Matter will continue to flow into the black hole but on a much longer viscous time scale. Such a mechanism would curtail the rapid growth of the black hole beyond a fraction of the core mass and thus limit the present observed mass of nuclear black holes. If there is a correlation between the initial properties of the core and the global properties of the galaxy, such as a correlation between core radius and effective radius of the spheroid (Faber et al. 1997), then the global scaling relations for SBHs– e.g., the mass-velocity dispersion relation– might be understood in the context of this mechanism.

The crucial question is whether or not constant density cores exist initially in spheroidal stellar systems. There are various observations consistent with the idea that the central density distributions are initially cusp-like and it is the orbital decay of massive binary black holes that creates a core– e.g., core galaxies are rounder and have reduced colour gradients (Lauer et al. 2005). On the other hand, we have seen that star formation expected in the rapid collapse of a central gas disc could produce both an apparent cusp in the density distribution and a colour gradient. It would appear that the issue of initial cores is not yet settled, but the mechanism discussed here could contribute to rapid fuelling of a black hole in any constant density triaxial core so long as it is not gravitationally dominated by the black hole. The simulations would, for example, also be relevant to the model of El-Zant et al. (2003) where cores are formed in a triaxial CDM halo during the initial baryonic collapse.

There are several possible sources for in-falling gas. One favoured possibility is that low angular momentum gas is supplied to the central region in merger events. The presence of counter-rotating discs in the central regions of some galaxies would seem to support this scenario. However, we have seen that a counter-rotating stellar disk can also be formed by the mechanism described here, even though the gas originally shares the rotation of the galaxy at large. This would give weight to a second possibility: low angular momentum gas can be supplied via mass loss from the stars in a slowly rotating system. Some of this gas is blown away in supernova heated winds, but some fraction of the gas may cool and flow into the centre, either simultaneously with a hot wind or in an unsteady cooling flow alternating with hot wind phases (Ciotti & Ostriker 1997). In either case– for an external or internal source–, the mechanism discussed here would bridge the final gap for transfer of gas from the core boundary to the black hole. It is a significant gap; gas in equilibrium at the core boundary must reduce its angular momentum by a factor of 10^{-8} to arrive at the Schwarzschild radius.

It has recently been discovered that the young stars observed within the central parsec of the Galaxy lie in two distinct disc systems at large inclinations to each other and with different senses of rotation (Paumard et al. 2006). The mechanism of dynamical collapse of gas discs would not apply here because the central parsec is gravitationally dominated by the 3×10^6 $M_\odot$ black hole; however, an in-falling gas disc in the presence of a dominant black hole (as in case *c* above), also bounces several times before settling into an accretion disk. It is evident in Fig. 5 that there is a large dissipation of energy in the first two bounces– a dissipation corresponding to strong compression. Thus it is possible that in the bounce star formation could proceed through strong shocks even in the near presence of the central black hole. This is a topic for future consideration.

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